Evolution of **G**

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Does microevolution predict macroevolution?

Can we predict species divergence (**D**) from within-population variance (**G**)?

Does divergence maintain its alignment with \mathbf{g}_{max} ?

What do we expect?

1. Reasons to expect alignment

2. Reasons to expect no alignment









Evolution of **G**

- Simple theory
- Simulations
- Data from Anolis
- Implications for macroevolution



Lande 1980, Phillips & Arnold 1989



Lande 1980, Phillips & Arnold 1989

$\Delta \mathbf{G} = \mathbf{G}(\boldsymbol{\gamma} - \boldsymbol{\beta} \boldsymbol{\beta}^{\mathrm{T}})\mathbf{G} + \mathbf{M} - \mathbf{R}$

Predictions of this model? Problems with this model?

Jones et al. G simulation models



https://github.com/JonesLabIdaho

Jones et al. model details

- Direct Monte Carlo simulation with each gene and individual specified
- Two traits affected by 50 pleiotropic loci
- Additive inheritance with no dominance or epistasis
- Allelic effects drawn from a bivariate normal distribution with means = 0, variances = 0.05, and mutational correlation $r_u = 0.0-0.9$
- Mutation rate = 0.0002 per haploid locus
- Environmental effects drawn from a bivariate normal distribution with mean = 0, variances = 1
- Gaussian individual selection surface, with a specified amount of correlational selection and ω = 9 or 49
- Each simulation run equilibrated for 10,000 (non-overlapping) generations, followed by several thousand of experimental generations

Mutation



Mutational effect on trait 1



Mutational effect on trait 1

Selection



Value of trait 1

Value of trait 1

Visualizing the **G**-matrix

$$\mathbf{G} = \begin{bmatrix} G_{11} & G_{12} \\ G_{12} & G_{22} \end{bmatrix}$$



Effect on the **G**-matrix



Stationary Optimum (selectional correlation = 0, mutational correlation = 0)



Stronger correlational selection produces a more stable **G**-matrix (selectional correlation = 0.75, mutational correlation = 0)



ω (trait 1)	ω (trait 2)	r (ω)	r (µ)	Δφ
49	49	0	0	9.1
49	49	0.25	0	9.2
49	49	0.50	0	8.9
49	49	0.75	0	7.8
49	49	0.85	0	5.4
49	49	0.90	0	4.3

A high correlation between mutational effects produces stability (selectional correlation = 0, mutational correlation = 0.5)



ω (trait 1)	ω (trait 2)	r (ω)	r (µ)	Δφ
49	49	0	0	9.9
49	49	0	0.25	7.9
49	49	0	0.50	3.6
49	49	0	0.75	1.5
49	49	0	0.85	1.1
49	49	0	0.90	0.9

When the selection matrix and mutation matrix are aligned, **G** can be very stable

90 60 30 φ 0 -30 -60 -90 Generations 2000 0 selectional correlation = 0.9, mutational correlation = 0.990 60 30 φ 0 -30 -60 -90 Generations 2000 0

selectional correlation = 0.75, mutational correlation = 0.5

What happens when the optimum moves?



Average value of trait 1

Peak movement along a genetic line of least resistance stabilizes the **G**-matrix



Average value of trait 1

Average value of trait 1

$\Delta \mathbf{G} = \mathbf{G}(\boldsymbol{\gamma} - \boldsymbol{\beta} \boldsymbol{\beta}^{\mathrm{T}})\mathbf{G} + \mathbf{M}$

Epistasis – The Multilinear Model

Multivariate extension of the multilinear model of Hansen and Wagner (2001):

Additive model:

$$z = a_1 + a_2 + a_3 + \ldots + e$$

Multilinear model:

$$z = a_1 + a_2 + \varepsilon a_1 a_2 + \dots + e$$

Only slightly more complex for the multivariate case

Epistasis Results

• Epistasis allows selection to shape the distribution of new mutations

Ν	r _w	V ₁	V ₂	r _v	M ₁	M ₂	r _µ
128	0	0.77	0.79	0.05	0.71	0.82	0.02
256	0	0.80	0.84	-0.02	0.46	0.50	-0.01
1024	0	0.70	0.70	0.00	0.17	0.16	-0.01
2048	0	0.68	0.65	-0.02	0.14	0.14	-0.01
128	0.90	0.33	0.32	0.41	0.43	0.42	0.08
256	0.90	0.33	0.33	0.45	0.24	0.26	0.14
1024	0.90	0.28	0.28	0.40	0.11	0.12	0.16
2048	0.90	0.26	0.27	0.40	0.10	0.10	0.18

Generations = 5000, ε variance = 1.0, ω = 49, μ = 0.0005











Conclusions from **G** matrix simulations













2845 juveniles, reared to 6 months









G matrix evolution

Matrices differ in size, shape, and orientation

Most matrices share no common PCs

Phylogenetic signal

Ecomorph effect

Is there a signature of genetic constraint in the *Anolis* radiation?



Common subspace analysis: How are G matrices most similar?

Tensor analysis: How do **G** matrices differ the most?











Common subspace analysis: How are G matrices most similar?

\mathbf{h}_1

avg % variance	52.8
jaw length	0.187
head width	0.139
pectoral	0.250
pelvis	0.197
humerus	0.500
ulna	0.491
femur	0.401
tibia	0.438

 h_1 explains 46 – 61% of variance within species

Similar to shared \mathbf{g}_{max} (92% of \mathbf{g}_{max})



d ₁	\mathbf{h}_1
73.5	52.8
0.017	0.187
0.365	0.139
0.140	0.250
0.216	0.197
0.323	0.500
0.532	0.491
0.373	0.401
0.522	0.438
	 d₁ 73.5 0.017 0.365 0.140 0.216 0.323 0.532 0.373 0.522

Most divergence is aligned with \mathbf{g}_{max}



e₁

% variance	40.7
jaw length	-0.020
head width	0.006
pectoral	-0.104
pelvis	-0.019
humerus	0.459
ulna	0.411
femur	0.531
tibia	0.571

84% of variation among **G** matrices in 3 subspaces ("eigentensors")

41% of variation explained by \mathbf{e}_1



	d ₁	\mathbf{h}_1	e ₁
% variance	73.5	52.8	40.7
jaw length	0.017	0.187	-0.020
head width	0.365	0.139	0.006
pectoral	0.140	0.250	-0.104
pelvis	0.216	0.197	-0.019
humerus	0.323	0.500	0.459
ulna	0.532	0.491	0.411
femur	0.373	0.401	0.531
tibia	0.522	0.438	0.571









-9 -

-9.5 + -4.5

-4

 $\log D_{G} = -1.35 + 1.83 \log G$ $R^{2}: 0.98$ -3.5 -3 -2 $\log G$

-2.5

Conclusions from Anolis work

General discussion